

**ACYCLICITY OF COMPLEMENTS OF  $\sigma$ -COMPACT WEAKLY  
INFINITE-DIMENSIONAL SUBSETS IN HILBERT CUBE**

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Acyclicity properties of subsets in the Hilbert cube are a subject of intensive study in infinite-dimensional topology. The complement of a compact, finite-dimensional subset in the Hilbert cube is acyclic. This result was generalised to compact  $C$ -spaces, and compact  $trt$ -dimensional spaces. The final generalisation of the above result was proved for weakly infinite-dimensional compact subspaces of the Hilbert cube. It is a well-known fact that the Hilbert cube cannot be made disconnected by removing a weakly infinite-dimensional (not necessarily closed) subspace. Some strongly infinite-dimensional compacta can separate the Hilbert cube. Further, a previous study proved that the complement of a weakly infinite-dimensional subset of the Hilbert cube is continuum connected. However, path-connectivity of the complement of a non-compact weakly infinite-dimensional subspace in the Hilbert cube is not established. In 2017, a result on the homology of complements of compact weakly infinite-dimensional spaces was proved. In particular, this showed that the complement of a weakly infinite-dimensional compact subspace of the Hilbert cube is path-connected since it has trivial 0-dimensional homology. In this research, we have generalised this result to complements of  $\sigma$ -compact weakly infinite-dimensional spaces. We prove that if  $X = \bigcup_{i=1}^{\infty} X_i$  is a  $\sigma$ -compact weakly infinite-dimensional subspace of  $Q$ , where each  $X_i$  is compact, then the complement of  $X$  has trivial 0-dimensional Steenrod homology. This argument is based on using Milnor's short exact sequence for Steenrod homology to the sequence of spaces  $K_i$ , which are closed tubular neighbourhoods of paths in the complement of  $X_i$ . Since there are continuum connected spaces with non-trivial 0-dimensional Steenrod homology, the above theorem is a generalisation of the continuum connectedness theorem proved in literature.

**Keywords:** Acyclicity, Hilbert cube, Steenrod homology, Weakly infinite-dimensional subspace